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Application of Probabilistic Analysis for Prediction of the Initiation of Landslide

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Abstract

Model for evaluating the failure probability of an inclined soil layer with an infinite length was developed in the present paper. Advantage of the probabilities analysis is since the model allows the uncertainties of soil properties, geology, and hydraulic properties of the slope. The main objective of the analysis is to estimate time of failure of slope by using Green-Ampt infiltration and infinite slope stability. Probability analysis is computed based on the Monte Carlo simulation method (MCSM). The model is then applied to evaluate the occurrence probability of shallow landslide-initiated debris flow in Tungmen gully located in the eastern Taiwan, which occurred a devastating debris flow in 1990. The statistical properties of hydrogeological parameters were collected and summarized. The soil parameter is assumed to be log normally distributed, while the hydraulic properties was assumed uniformly distributed. The simulation results the mean time to failure (MTTF) and standard deviation are about 14 hours and 3.2 hours respectively. The failure probability of MTTF is about 0.54 to occur at 11am on 23 June 1990. The failure rate of "degree of belief" for occurrence at the estimated time is calculated 0.27.

1. INTRODUCTION

It is widely recognized that slope stability analysis is characterized by numerous uncertainties due to limited sampling, discrepancy between different methods of laboratory and in situ strength testing, and uncertainties in soil models. Considering the uncertainty of parameters, Muntohar [1] have developed an integrated rainfall infiltration and slope stability model for evaluating the landslide occurrence, which is related to a failure probability (P_f) for slope with an infinite length, based on the monte carlo simulation method (MCSM). The model was extended from the deterministic model by Muntohar and Liao [2]. The developed model can be applied to estimate initiation and failure probability of landslide. Chen and Jan [3] built probabilistic model for evaluating the occurrence probability of landslide related debris flow. However, the model is limited to the condition of a

saturated soil layer having the water table higher than the soil layer surface. The model was applied in Tungmen landslides in 1990 which located in the eastern Taiwan.

In some cases, landslides could occur in an unsaturated soil layer when the water table lies below the surface of the soil layer. Thus an extended probability-based model for evaluating the stability of an inclined soil layer having either water table below or above the soil-layer surface is developed in the present study. A model should be built by considering the effect of rainfall infiltration. In this paper, a rainfall infiltration and infinites slope stability model is presented based on the probability analysis. The paper is addressed to overcome the uncertainties in slope failures during heavy rainfall.

2. METHOD OF ANALYSIS

2.1 Rainfall Infiltration and Slope Stability Model

To model a rainwater infiltration induced shallow slope failure, a relationship between rainwater infiltration and slope stability should be established. Infiltration model was modified from Green and Ampt model. Originally the model was developed for horizontal ground surface. The model is illustrated in Figure 1. In the Green and Ampt model (Figure 1a), the suction head at the wetting front is assumed to be constant, and in the zone of soil above the wetting front, the moisture deficit (the difference between the volumetric water contents before and after wetting) is uniform, and the coefficient of saturated hydraulic conductivity is constant. This means that the soil is fully saturated from the surface to the depth of the wetting front, while the soil below the wetting front is at the initial degree of saturation. Application of the Green-Ampt model requires estimate or measurement of the hydraulic conductivity (k), the porosity (η) or the saturated water content (θ_s), the initial water content (θ_i), and the wetting front soil suction (ψ_f). In term of unsaturated-saturated state, these parameters are correlated among others.

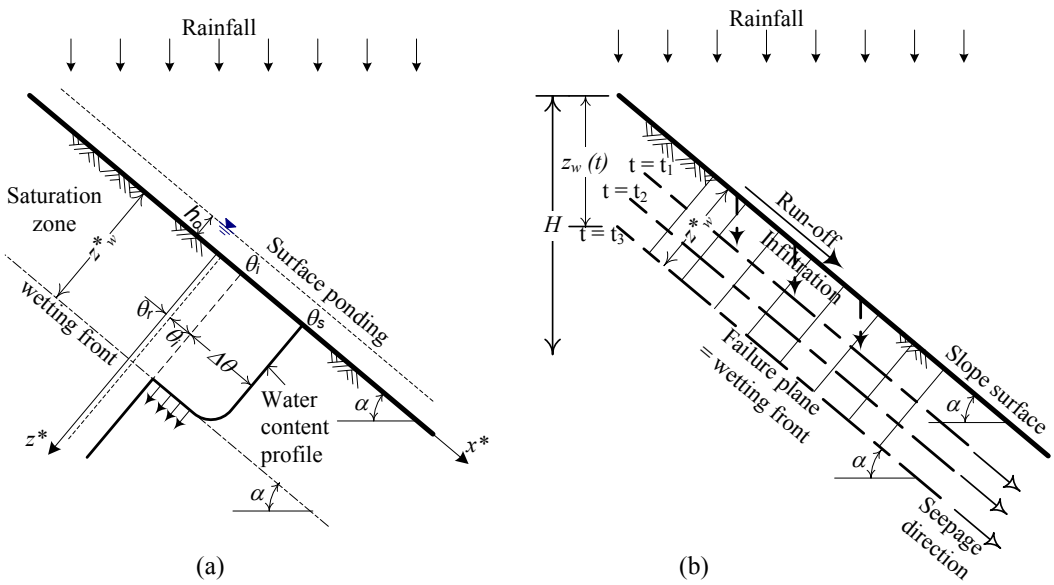

Fig. 1 Rainfall infiltration model on infinite slope.

The depth of wetting front (z_w) is obtained from the relationship given in Equation (1) or (2). For the sloping ground (see Figure 1b), thus, the depth of wetting front perpendicular to slope surface at time t is written in Equation (1).

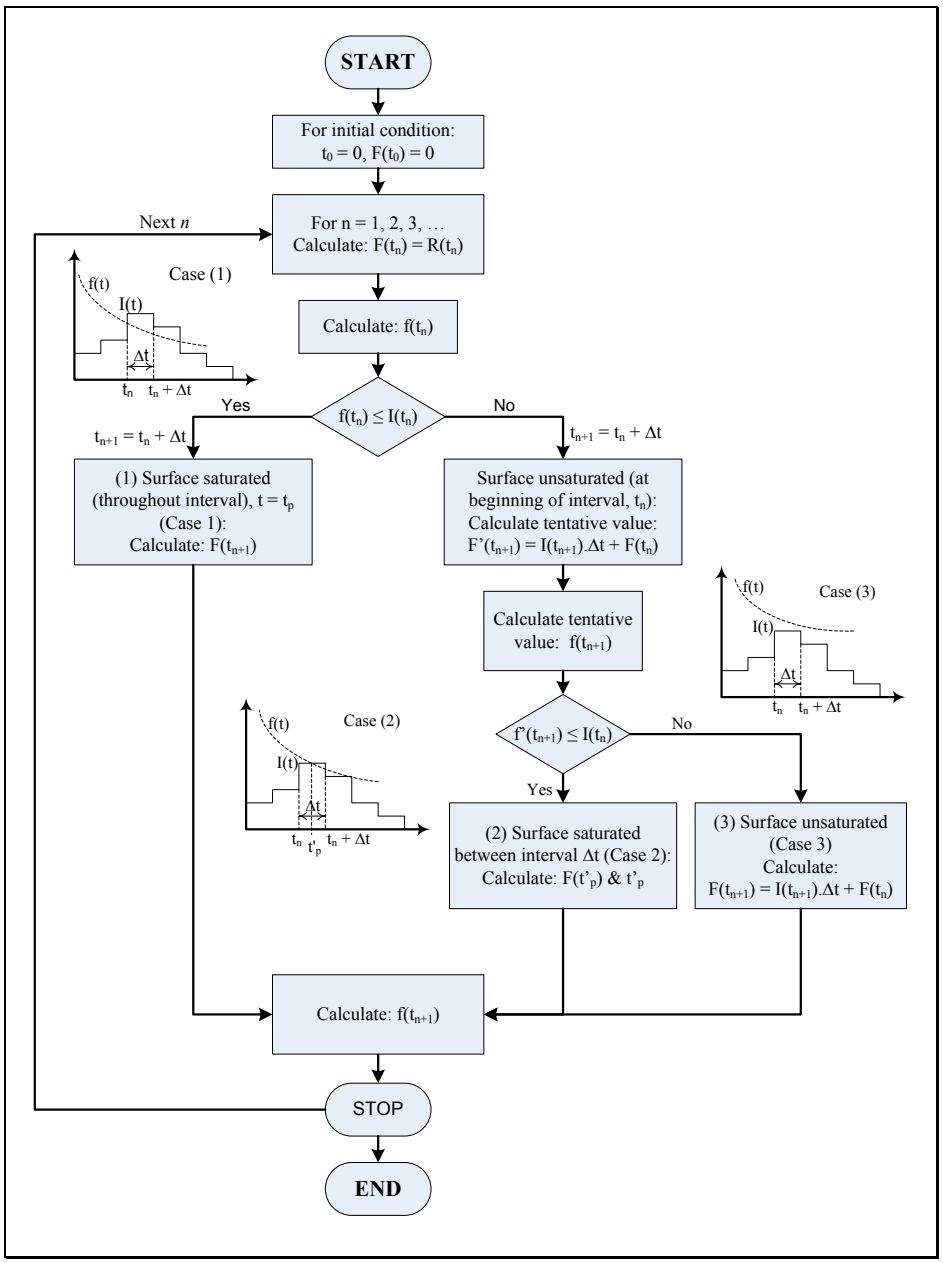

Fig. 2 Flowchart for determining infiltration under unsteady rainfall intensity by using Green-Ampt model (Modified from Chow et al. [4]).

$$z_w{}^*(t) = \frac{F(t)}{\Delta\theta} \tag{1}$$

Since the depth of wetting front change with the elapsed time, the calculated factor of safety will also change with the elapsed time of rainfall. Figure 2 presents the flowchart adopted to calculate the Green-Ampt infiltration and slope stability model in this study.

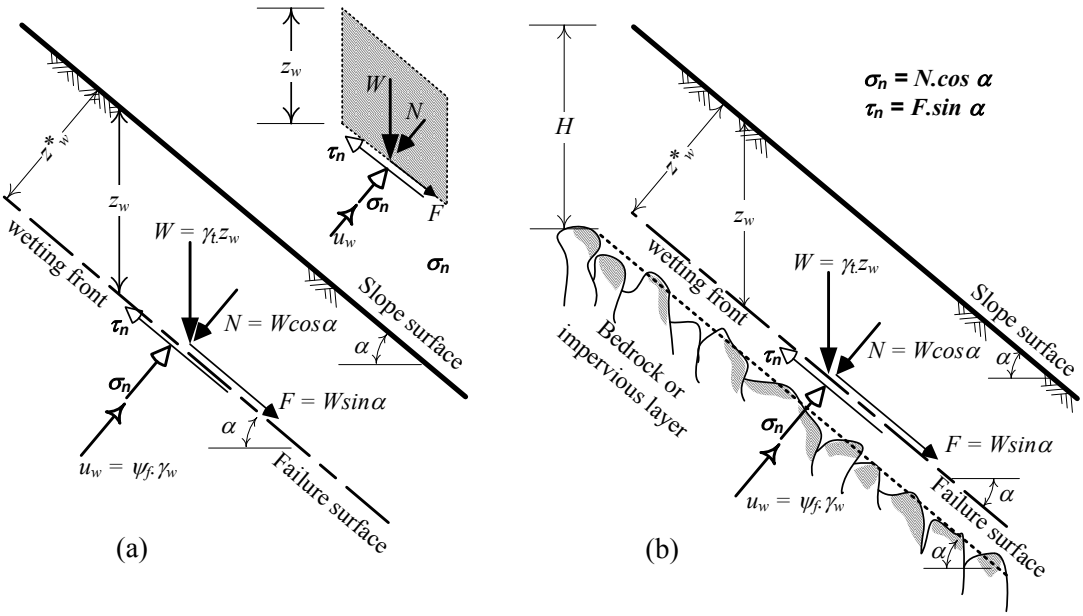

Fig. 3 Infinite slope model (a) Model 1: bedrock or impervious layer is not presence or very deep (b) Model 2: bedrock or impervious layer is presence at very shallow depth.

In this study, the infinite slope is distinguished into 2 models as shown in Figure 3. The first model (Model-1, Figure 3a) assumed that the presence of bedrock or impervious layer is very deep or does not exist. While the second model assumed that the bedrock or impervious layer exists at shallow depth. For this model, the thickness of soil is shallow and thin. The critical slip surface is assumed to occur at the wetting front and the interface of soil and bedrock or impervious layer. Thus, by considering the unsaturated soil behavior, the factor of safety (FS) against an infinite slope sliding can be calculated as follows:

- Model-1 (Figure 3a), the equation is suggested by Cho and Lee [5]:

$$FS = \frac{c' + \left(\gamma_t z_w \cos^2\alpha - u_w\right)\tan\phi'}{\gamma_t z_w \cos\alpha \sin\alpha} \tag{2}$$

Crosta [6] suggested that the pore water pressure (u_w) is maintained constant, $u_w = \psi_f \cdot \gamma_w$, since the water table is very deep or does not exist. While the suction head at wetting front (ψ_f) can be determined from soil water characteristic curve (SWCC).

- Model-2 (Figure 3b):

$$FS = \begin{cases} \dfrac{c' + \left(\gamma_t z_w \cos^2 - u_w\right)\tan\phi'}{\gamma_t z_w \cos\alpha \sin\alpha} & ; \text{for } z_w < H \\ \dfrac{c' + \left(\gamma_t H \cos^2\alpha - u_w\right)\tan\phi'}{\gamma_t H \cos\alpha \sin\alpha} & ; \text{for } z_w \geq H \end{cases} \tag{3}$$

where c' and ϕ' are soil cohesion (kPa) and internal friction angle of soil (_°) respectively; γ_t is the saturated unit weight of soil (kN/m^3); γ_w is the unit weight of water (= 9.81 kN/m^3); α is the slope angle (_°); z_w is the wetting front depth (m); and H is the depth of bedrock or impervious layer (m). For $z_w < H$ in Equation (3) is in the same form as the Equation (2).

2.2 Parameter uncertainties, reliability and failure probability

In the limit equilibrium based infinite slope stability analysis, the slope failure will occur when the factor of safety is below one (FS < 1), and the critical state is obtained when the factor of safety is equal to one (FS = 1). The performance function can be obtained from Equation (3) as follows:

$$G(X) = \frac{c' + \left(\gamma_t z_w \cos^2\alpha - u_w\right)\tan\phi'}{\gamma_t z_w \sin\alpha \cos\alpha} \tag{4}$$

The performance function involves seven parameters: c', ϕ', γ_t, α, H, γ_w, and z_w. The last parameter involves three other parameters in infiltration analysis: k_s, ψ_f, and $\Delta\theta$. Two parameters γ_w and α are treated as the deterministic variables since the α and γ_w are easily to measure and the variability is small. The others parameters are treated as the uncertain variables, X_i = {c', ϕ', γ_t, α, H, k_s, ψ_f, $\Delta\theta$}. These variables are sampled randomly from their probability density distribution which is assumed as identically-independent distribution (i.i.d).

The computed factor of safety in Equations (2) and (3) varies with the elapsed time. For a given parameter in the equations, a slope will fail if the FS < 1, otherwise the slope is stable if the FS > 1. Thus, the indicator function (I_F) to distinguish failed and not-failed can be expressed as

$$\mathbb{I} \quad \begin{cases} & : FS > 1 \\ & : FS < 1 \end{cases} \text{(indicator function)} \tag{5}$$

In this study, Monte Carlo Simulation Method (MCSM) was performed to obtain the failure probability. Values of each uncertain variable are generated randomly as i.i.d. from the probability distribution function (PDF) for each N simulation cycles. The failure probability from MCSM can be approached by calculating the following equation:

$$P_f^{MCS} = E\left(\mathbb{I} \quad \mathbb{I} \quad ..,\alpha^{(i)}\right) \tag{6}$$

A slope is likely to fail if the FS ≤ 1. The time which corresponds to the FS = 1 is defined as the time of failure or time of landslide initiation (T_f). Using the MCSM, the distribution of failure time

can be plotted. Then, the time of failure can be estimated from the statistical properties of its distribution function.

$$\psi_{T_f} = \frac{\sum_{i=1}^{k} N_{FS\min} \cdot t_i}{N_f} \tag{7}$$

$$\zeta_{T_f} = \sqrt{\frac{\sum_{i=1}^{k} N_{FS\min} \cdot \left(t_i - \mu_{T_f}\right)^2}{N_f}} \tag{8}$$

where N_{FSmin} is the number of samples which fail at time t; N_f is the number of accepted samples; μ_{T_f} and ζ_{T_f} are the mean time to failure (MTTF) and standard deviation of the time of failure.

2.3 Rate of Failure

The failure rate is defined for non repairable populations as the (instantaneous) rate of failure for the survivors to time t during the next instant of time. The next instant the failure rate may change and the units that have already failed play no further role since only the survivors count [7].

The failure rate (or hazard rate) is denoted by $h(t)$ and calculated from

$$h(t) = \frac{f(t)}{1 - F(t)} \tag{9}$$

where $f(t)$ and $F(t)$ are probability density function and cumulative density function of the model respectively. It is also sometimes useful to define an average failure rate over any interval (T_1, T_2) that "averages" the failure rate over that interval.

3. APPLICATION

A landslide occurred in Tungmen village, Hualien, Taiwan during typhoon Ofelia on 22-23 June 1990 as shown in Figure 4. Typhoon Ofelia tracked from the southeast to the northwest of Taiwan from 22 to 23 June 1990. This violent storm started unleashing heavy rainfall on the Tungmen area at 02:00pm of 22 June which totaled 160 mm by 7 AM on 23 June (Figure 5a). The rainfall restarted at 8 AM and ceased at 4 PM on 23 June, finally the amount of rainfall was 450 mm. The debris flow began to appear on the afternoon of 23 June. Nearly half of the village was covered by the debris flow having a volume of about 55,000 m^3 [8, 9]. The slope was covered by a soil layer up to 4 m. The soil layer was majority sandy gravel which is classified into GP-GM, GM, and GM-GC [10]. The sediment layer of the gully bed was up to 2 m during the 1990 debris-flow event. The slope and geotechnical properties of the studied area are presented in Table 1. The soil parameter is assumed to be log normally distributed, while the hydraulic properties was assumed uniformly distributed.

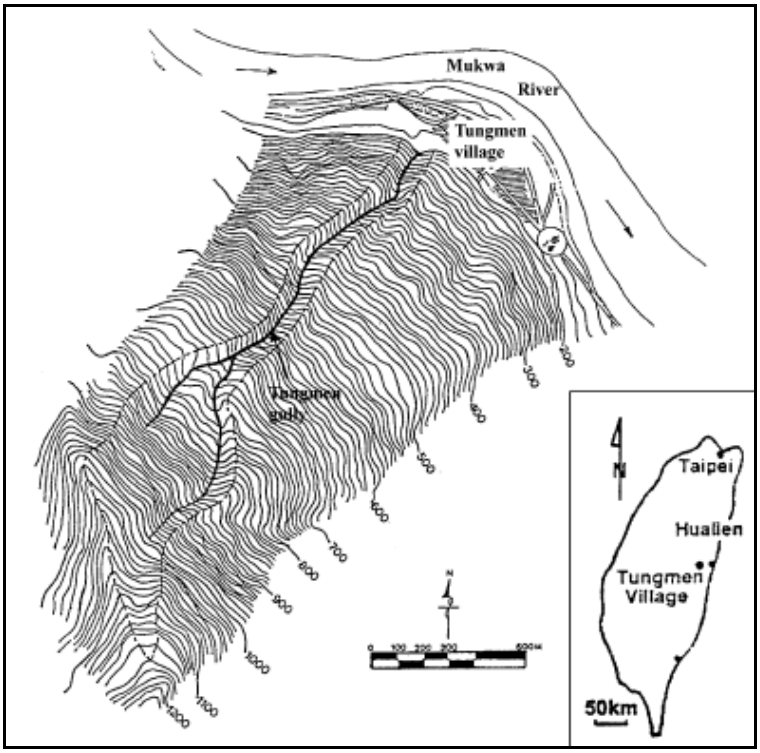

Fig. 4 Location and watershed of a debris-flow gully behind Tungmen village in Hualien, Taiwan (Chen & Jan, 2008).

Table 1 Parameters used for Tungmen landslide cases.

Slope angle α	Hydraulic conductivity k_s (mm/h)	Moisture difference $\Delta\theta = \theta_s - \theta_i$	Wetting front suction head ψ_f (mm)	Cohesion c' (kPa)	Friction angle ϕ'	Soil unit weight γ_t (kN/m^3)	Bedrock Depth H (m)
20° (0.05)	0.36 - 360	0.125 (0.046)	3.2 (3.71)	0	34.5^o (0.05)	20.6 (8.95)	3.5 (1.51)

Note: The values are mean values. Numbers in bracket is the standard deviation values.
H = depth of bedrock or impervious layer. The ψ_f is estimated from particle size distribution using Arya and Paris method [11]. The k_s is assumed to be uniform distribution.

4. RESULTS AND DISCUSSIONS

4.1 Failure Probability and Failure Rate

To understand the failure mechanism, Figure 5 shows the computation result including the estimated depth of sliding surface (Figure 5b) and failure probability of a given time (Figure 5c

and 5d). Assuming the depth of sliding surface occurred at the depth of wetting front, the mean value of wetting front depth (μ_{ψ_f}) reaches 2.2 m to 2.4 m deep as shown in Figure 5b. It indicates that soil layer above gully bed have been washed-away during the rainfall event. During the rainfall, considering the variance ($\sigma^2_{\psi_f}$)with elapsed time, the wetting front be able deepening up to 3.9 m deep below the slope surface as shown in Figure 5b. An investigation carried out by Chen et al. [8] found that the gully bed have been deepened about 5 m after the debris flow.

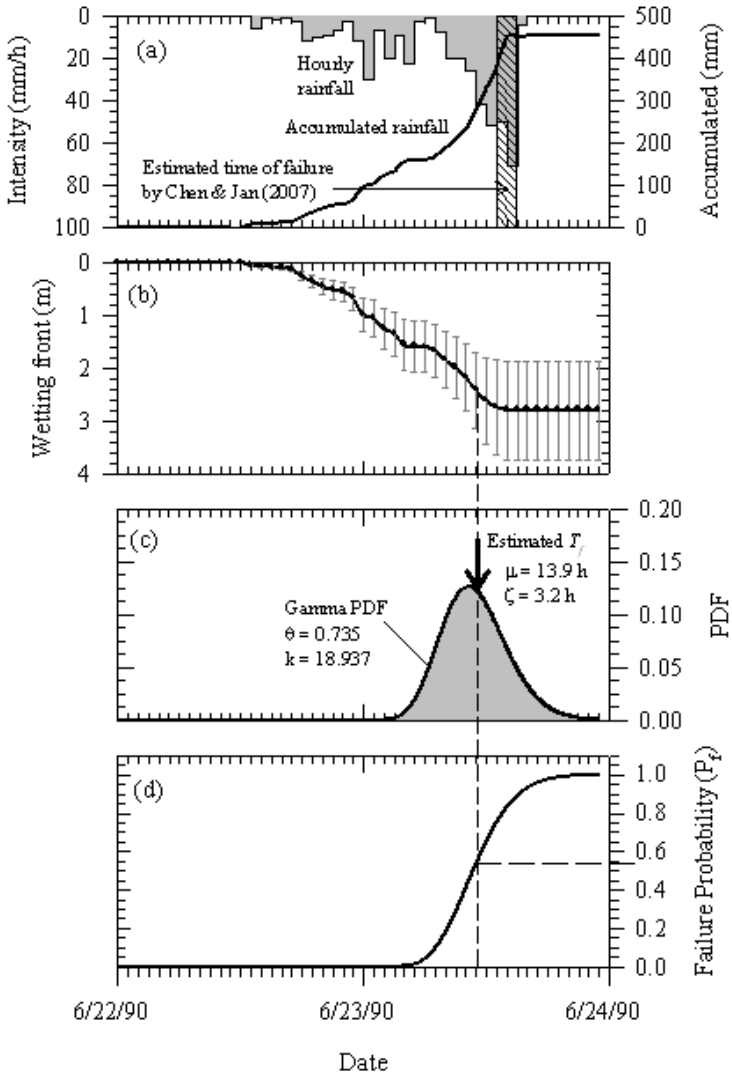

Fig. 5 Slope failure analysis at Tungmen, Hualien, Taiwan (a) hourly rainfall and accumulated rainfall, (b) computed depth of wetting front, (c) computed failure probability, (d) distribution plot of the time of failure.

Based on the MCSM, the probability density function of time of failure is presented in Figure 5c. The time of failure is approached using shifted gamma model. The mean time to failure (MTTF) and standard deviation are about μ_{T_f} = 14 hours and standard deviation ζ_{T_f} = 3.2 hours. Considering the variance, the time of failure corresponds to the failure probability ranging from 0.45 to 0.99 as shown on Figure 5d. The failure probability of MTTF is about 0.54 to occur at 11am on 23 June 1990. It implies that landslide is potentially to occur about 24 hours after the beginning of typhoon. The estimated time of failure is 3 hours earlier from the recorded by Chen [9]. This result indicates that the landslide is initiated at 11am and then following debris flow occurs at 2pm.

Calculation of the failure rate can be denoted, in this case, as "degree of belief" of time to failure. A higher value of failure rate indicate a higher "degree of belief" that the landslides will take place at a given time *t*. Figure 6 shows the failure rate and frequency from the simulation. The failure rate at MTTF is calculated about 0.27 per hour.

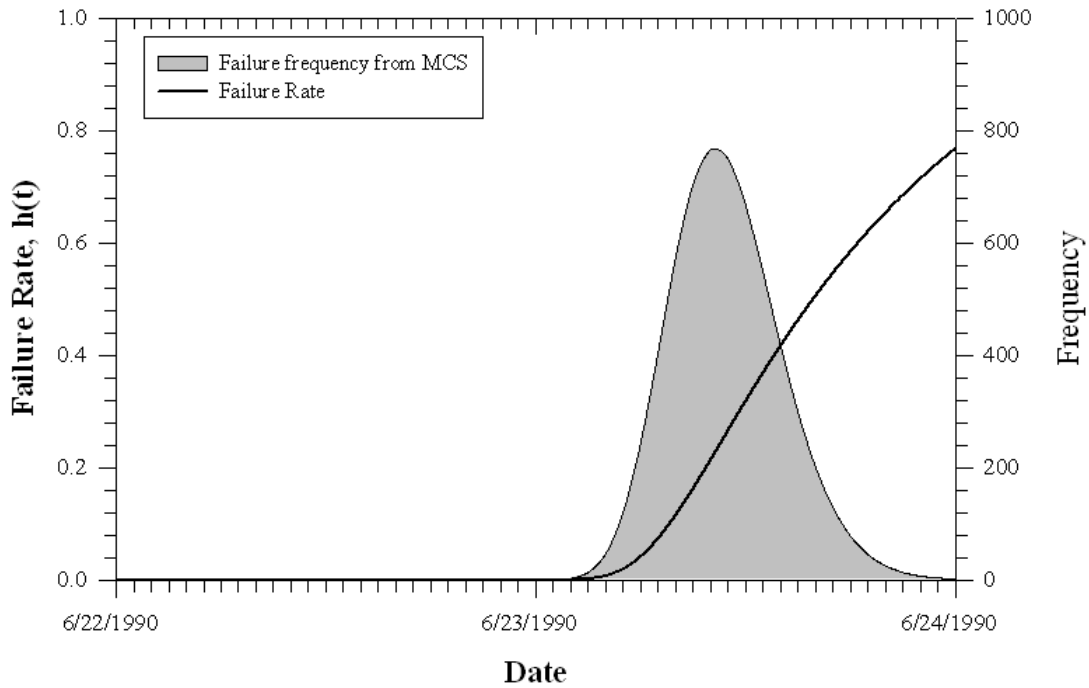

Fig. 6 Failure rate and frequency from the MCS.

4.2 Critical Rainfall

Based on the characteristic, one can estimated the triggering rainfall induces landslide. Use Figure 5a, the critical rainfall intensity and accumulated rainfall induced landslides are 42 mm/h and 275 mm respectively. The rainfall threshold is laid above the empirical threshold proposed by Chen et al [12] as shown in Figure 7. In cases of debris flow, the time taken cumulative runoff, to yield abundant water for debris triggering, is an important index that needs monitoring.

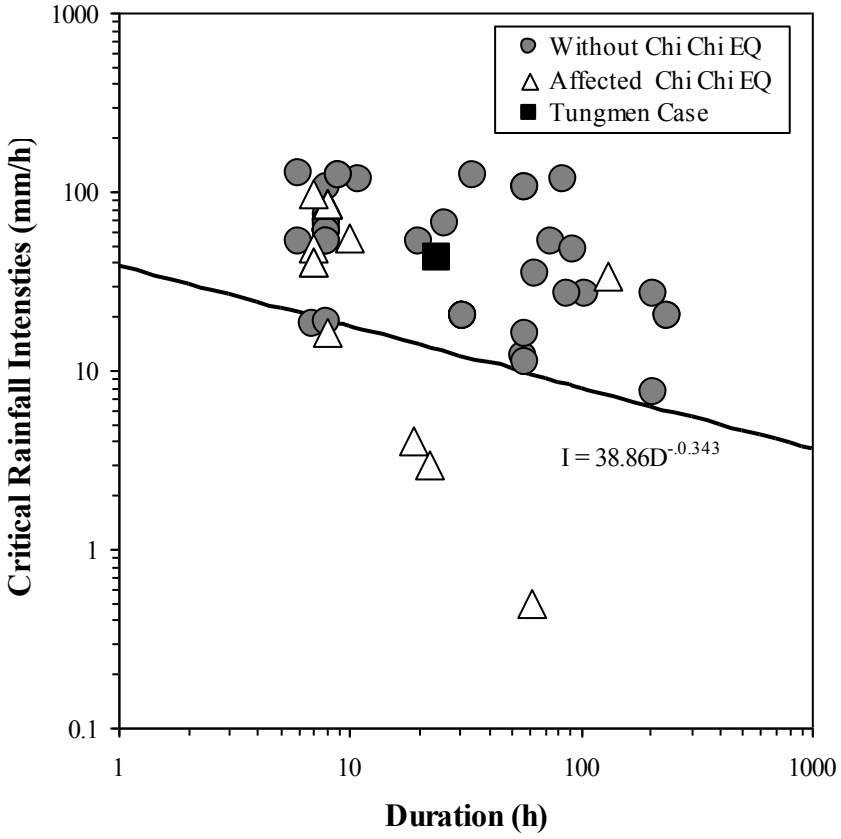

Fig. 7 Critical rainfall intensity–duration graph

5. CONCLUSIONS

The research has been successfully carried out for estimating the initiation of landslide occurrence based on the probabilistic method. Applying the probabilistic model, one needs to know then statistical properties (mean values and coefficients of variation) of soil properties, geology, and hydraulic parameters in advance, such as c', ϕ', γ, α, H, k_s, ψ_f, and $\Delta\theta$. Some conclusions can be drawn from the analysis and discussion as following:

a. The time of failure is approached using shifted gamma model. The mean time to failure (MTTF) and standard deviation are about μ_{T_f} = 14 hours and standard deviation ζ_{T_f} = 3.2 hours. The failure probability of MTTF is about 0.54 to occur at 11am on 23 June 1990, which is 3 hours earlier from the reported and following by debris flow at 2am.
b. The failure rate at MTTF is calculated about 0.27 per hour.

c. The critical rainfall intensity and accumulated rainfall induced landslides are 42 mm/h and 275 mm respectively

6. ACKNOWLEGEMENT

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